Actuator fault estimation based on a switched LPV extended state observer

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Abstract

Actuator fault estimation problem is tackled in this paper. The actuator faults are modeled in the form of multiplicative faults by using effectiveness factors representing the loss of efficiency of the actuators. The main contribution of this paper lies in the capacibility of dealing with the presented problem by using a switched LPV observer approach. The LTI system in the presence of faulty actuators is rewritten as a switched LPV system by considering the control inputs as scheduling parameters. Then, the actuator faults and the system states are estimated using a switched LPV extended observer. The observer gain is derived, based on the LMIs solution for the switched LPV systems. The presented actuator fault estimation approach is validated by two illustrative examples, the first one about a damper fault estimation of a semi-active suspension system, and the second one concerned to fault estimations on a multiple actuators system.

Key words: Actuator fault estimation, switched LPV observer, dwell time.

1 Introduction

Fault estimation is a step in Fault Diagnosis and plays a key role in designing a fault tolerant control. Many different approaches have been developped to estimate a fault which can be either actuator or sensor malfunction. Let us mention some classical methods, based on the parity space theory [10] to generate the residues and approximate the fault, or the bank of observers approach [14], or the sliding mode observers [8].

Recently, [12] used an Unknown Input Proportional Integral Observer for actuator fault detection and estimation but this method is more oriented for constant faults. To deal with the time-varying faults, [23] presented a method using a Fast Adaptive Fault Estimation (FAFE) methodology based on an adaptive observer. Therein, the authors solved the problem with a regular LTI system without considering the disturbances. Then, [19] proposed an adaptive polytopic unknow input observer for time-varying fault estimation, for a class of descriptor LPV systems.

Besides, several works have been done for fault estimation and fault tolerant control for LPV systems.[1] proposes the design of a low-order LPV observer to estimate the unmeasured states of the system and to estimate the sensor faults for a class of uncertain LPV systems. An LMI-based pole-placement robust LPV estimator is presented in [18], an interval observer for LPV systems in [17], an LPV sliding mode observer in [11]. Moreover, a new approach in [20] considers the fault element as a state of the augmented system and an LPV extended observer is designed to estimate, at the same time, the state and the fault of system where constant faults are considered.

Regarding to the actuator fault estimation problem, along with the additive model, the fault can be also written in a multiplicative form by using fault effect factors which are assumed to be constant or slow-varying. Besides, inspired by the fact that the control input is known, the considered system can be rewritten as an extended LPV system while considering the control inputs as scheduling paramters. Moreover, it has to be noticed that when the control input is zero, the actuator fault information is not available. Therefore, to deal with this problem, the considered system is modeled as a switched LPV system .

It is well known that for the switched system, the main challenge for a controller or observer design problem for this type of system is to deal with the stability analysis, especially for the continuous time switched system due to the discontinuities of the Lyapunov function at switching instants. A lot of studies have been investigated for such a problem during recent decades. Let us mention firstly the LTI case. The stability of continuous-time LTI switched systems has been addressed in [5],[13], [15], [9]. Therein, the stability under dwell time, average dwell time constraints using multiple Lyapunov functions are studied. The main idea is to ensure

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that the Lyapunov function is non-increasing at switching instants or to allow some short-time increasing but the dwell time between two consecutive switching instants is sufficient large in order to compensate for possible increase of Lyapunov functions. In particular, [9] provides an efficient way to get the stability for the switched LTI system under minimum dwell-time by mean of a family of quadratic Lyapunov function.

However, such an approach is not easy to extend to the switched uncertain systems because of nonconvex dependence on the system matrices. To overcome this, recently, [7] presented the Lyapunov looped-functionals approach, a new type of functionals leading to stability conditions that are affine in the system matrices. [6] proposes an alternative solution using lifted conditions which are convex in the system matrices and shown to be equivalent to the nonconvex conditions proposed in [9]. Another possibility is presented in [2], [3] where a piecewise linear in time, quadractic Lyapunov form function is used to derive convex conditions for both stability and stabilization problems with dwell time.

For the class of switched LPV systems, most works have considered the control problem but very few the observation one. [16] proposed a switching LPV controller based on the multiple Lypunov functions under hysteresis switching and average dwell time constraint. [24] dealt with a model reduction problem for switched LPV system in which the stability was derived also by using the average dwell time technique.

The main purpose of this paper is to propose a methodology to estimate multiplicative faults for the actuators. An actuator time-varying fault is considered in the form of actuator power loss. Then, effectiveness factors are used to model the efficiency of actuators. The paper contributions emphasize on the following aspects. Firstly, the system in the presence of actuator faults is modeled in the form of a switched LPV system by considering the control inputs as scheduling parameters. Secondly, a switched LPV extended observer is designed to estimate both the actuator faults and the system state. The stability of the switched LPV observer will be guaranteed using the dwell time constraint. To this aim, a non-increasing piece-wise linear in time Lyapunov function is assigned for each subsystem. The $\mathscr{H}_\infty$ performance is used to minimize the $\mathscr{L}_2$ gain from the disturbance to the estimation error.

The rest of this paper is organized as follows: the next section presents the problem formulation. Section 3 gives some preliminaries for this work. Section 4 gives a full description for multiplicative faults estimation based on the switched LPV observer approach. In Section 4, two numerical examples are presented to illustrate the effectiveness the proposed approach. Finally, some conclusions are drawn in the section 5.

2 Problem Formulation

2.1 System definition

Consider a continuous time linear invariant system:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_2u(t) + B_1w(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, w(t) \in \mathbb{R}^q$ and $y(t) \in \mathbb{R}^p$ are the state, the control input, the input disturbance and the measured output vectors, respectively. Matrices $A \in \mathbb{R}^{n\times n}, B_2 \in \mathbb{R}^{n\times m}, B_1 \in \mathbb{R}^{n\times q}, C \in \mathbb{R}^{p\times n}$ are known matrices of appropriate dimensions.

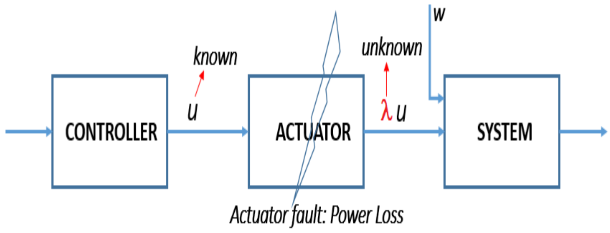

Fig. 1. System with actuator fault

In this work, only actuator faults are considered. Assume that the system (1) is in the faulty actuator situation, e.g loss of actuator power (Fig.1). These actuator faults are modeled in a multiplicative representation. In fact, denoting that $\bar{u}_i$ is the output of i^{th} faulty actuator , then:

$$\bar{u}_i(t) = \lambda_i u_i(t) \tag{2}$$

where λ_i stands for the efficiency coefficient of the i^{th} actuator and λ_i is assumed to be constant. $\lambda_i = 1$ implies that the i^{th} actuator is fault-free, $\lambda_i = 0$ mean that the i^{th} actuator is in total failure. And $0 < \lambda_i < 1$ represents the fact that the fault of the ith acutator is a partial loss of control effectiveness, e.g if $\lambda_i = 0.8$, the i^{th} actuator loses 20% of its effectiveness.

It is worth noting also that even if λ_i is assumed to be constant, the corresponding additive fault magnitude on ith actuator given by $f_i(t) = (1-\lambda_i)u_i(t)$ is a time varying signal and depends on the value of the control input $u_i(t)$. Thanks to the multiplicative representation, the information about the actuator fault λ_i is considered as constant or slow-varying and $\dot{\lambda}_i = 0$ which will be used later for the extended system.

In the presence of the actuator faults, the input matrix B_2 becomes $B_2\Lambda$ where $\Lambda \in \mathbb{R}^{m\times m}$ is a diagonal matrix representing the impact factors of the actuator faults, i.e:

$$\Lambda = diag([\lambda_1\ \lambda_2 ... \lambda_m])$$

Then, the following equation gives the representation of the faulty system subject to the actuator faults:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_2\Lambda u(t) + B_1w(t) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

Since Λ is a diagonal matrix, (3) can be rewritten as:

$$\dot{x}(t) = Ax(t) + B_2 \underbrace{\begin{bmatrix} u_1 & & & \\ & u_2 & & \\ & & \ddots & \\ & & & u_m \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \end{bmatrix}}_{\lambda} + B_1 w(t) \tag{4}$$

The objective of this work is to estimate the vector $\lambda = [\lambda_1\ \lambda_2\ ...\ \lambda_m]^T$ where λ_i represents the effectiveness factor of the actuator i^{th}. The estimation, based on an extended switched observer, is presented in the sequel.

2.2 Switched LPV system

Thanks to the multiplicative fault representation, considering $\dot{\lambda}_i = 0$, the system (4) can be augmented as follows:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & B_2 U \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w \\ y = [C \quad 0] \begin{bmatrix} x \\ \lambda \end{bmatrix} \end{cases} \tag{5}$$

where the dimension of the augmented system is $n+m$.
In order to estimate the vector λ, the system (5) must be observable or at least detectable. However, it can be seen that if the control input $u_i(t) = 0$, then the fault information of the ith actuator λ_i in (5) becomes unobservable. It makes the problem unfeasible in the observer design step. Thus, in this proposed method, the persistent excitation condition on the control input u is required. Moreover, an interesting remedy is to take into account the effect of the sign of $u_i(t)$ in the observer synthesis step, i.e the observer will be designed for different cases where the control inputs are positive and negative. To account for the change the sign of $u_i(t)$, the system will be rewritten as a switched system and a switched observer will be designed in the sequel.It is well known that in a real mechatronic system, the actuators always admit some physical constraints, i.e the control input $u(t)$ satisfies the following saturation constraint:

$$u(t) \in \mathscr{U} = \left\{ u \in \mathbb{R}^m | \underline{u_i} \leq u_i \leq \overline{u_i} \right\} \tag{6}$$

where $\underline{u_i}, \overline{u_i}$ are the lower, upper bounds of the ith actuator u_i.
Moreover, since the control input $u(t)$ is known, the system (3) can be represented as an LPV system by choosing $u(t)$ as the vector of scheduling parameters. Let us rewrite $u_i(t) = |u_i(t)| sign(u_i(t))$, and denote $\rho_i(t) = |u_i(t)|$ as a scheduling parameter. Then,

$$u_i(t) = \rho_i(t) sign(u_i(t)) = \begin{cases} \rho_i(t) & if \quad u(t)) \geq 0 \\ -\rho_i(t) & if \quad u(t) < 0 \end{cases} \tag{7}$$

Thus, the scheduling parameter vector $\rho = [\rho_1\ \rho_2\ ...\ \rho_m]^T$ is assumed to read for the following constraint:

$$\rho(t) \in \Omega = \{ \rho \in \mathbb{R}^m | \varepsilon \leq \rho_i \leq \overline{\rho_i} \} \tag{8}$$

where $\varepsilon > 0, \overline{\rho_i}$ are the bounds of the parameter ρ_i.
The faulty system (4) is now rewritten as the following LPV system:

$$\dot{x}(t) = Ax(t) + B_\sigma(\rho)\lambda + B_1 w(t) \tag{9}$$

where $B_\sigma(\rho) = B_2 \begin{bmatrix} \rho_1 \text{sign}(u_1) & & & \\ & \rho_2 \text{sign}(u_2) & & \\ & & \ddots & \\ & & & \rho_m \text{sign}(u_m) \end{bmatrix}$
and $\lambda = [\lambda_1\ \lambda_2\ ...\ \lambda_m]^T$.

The system (9) can then be augmented into the following form:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \underbrace{\begin{bmatrix} A & B_\sigma(\rho) \\ 0 & 0 \end{bmatrix}}_{A_e(\rho)} \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w \\ y = \underbrace{[C \quad 0]}_{C_e} \begin{bmatrix} x \\ \lambda \end{bmatrix} \end{cases} \tag{10}$$

The system (10) is actually a switched system where $\sigma(t)$ is the switching rule that depends on the value of the function $\text{sign}(u_i(t))$ and takes values on the discrete set $\{1, 2, ..., M\}$. $M = 2^m$ is the number of "modes" that compose the overall switched dynamics. Indeed, the matrix $B_\sigma(\rho)$ switches between different matrices and the switching moments depend on the sign of the actuators $\text{sign}(u_i(t))$. For example, if we consider a system with 2 actuators ($m = 2$), the matrix $B_\sigma(\rho)$ belongs to the following set:

$$\left\{ B_2 \begin{bmatrix} \rho_1 & \\ & \rho_2 \end{bmatrix}, B_2 \begin{bmatrix} -\rho_1 & \\ & \rho_2 \end{bmatrix}, B_2 \begin{bmatrix} \rho_1 & \\ & -\rho_2 \end{bmatrix}, B_2 \begin{bmatrix} -\rho_1 & \\ & -\rho_2 \end{bmatrix} \right\}$$

Therefore, we can rewrite the system (10) in the following switched LPV system form:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = A_{e,\sigma}(\rho) \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w \\ y = C_e \begin{bmatrix} x \\ \lambda \end{bmatrix} \end{cases} \tag{11}$$

where $A_{e,\sigma}(\rho) \in \mathbb{R}^{(n+m)\times(n+m)}$ switches among the subsystems $\{A_{e,1}(\rho), A_{e,2}(\rho), ..., A_{e,M}(\rho)\}$.

2.3 Problem Statement

The actuator fault estimation problem is recast into that of the parameter vector λ. To this aim, a switched LPV Extended State Observer (ESO) will be designed for the switched LPV system (11).
Then, the following detectability condition is assumed for the design of the ESO of extended LPV system (11):

$$\text{A1: } rank\left(\begin{bmatrix} sI - A_{e,\sigma}(\rho) \\ C_e \end{bmatrix}\right) = n+m, \quad s>0, \forall \rho \in \Omega \quad (12)$$

Remark 1 *The observability for the LPV system is not a trivial problem because the variation of the parameter $\rho(t)$ is considered in computing the observability matrix. Several definitions of observability for the LPV systems have been given in the literature such as Quadratic detectability [22] in the sense of Lyapunov function and Structural Observability [21] i.e the observability matrix is full rank in function sense of $\rho(t)$ but one can lose the observability in some frozen point.*
Under the Assumption A1, the problem consists now in designing a switched LPV observer in order to estimate the effectiveness factors of the actuator faults. Therefore, considering the switched LPV system (11), the following switched LPV extended observer is proposed to estimate the system's state and the effectiveness coefficient vector λ :

$$\begin{cases} \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\lambda}} \end{bmatrix} = A_{e,\sigma}(\rho)\begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix} + K_\sigma(.)(y-\hat{y}) \\ \hat{y} = C_e \begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix} \end{cases} \quad (13)$$

From (11) and (13), the estimation error $e(t)$ is calculated by:

$$\dot{e} = \begin{bmatrix} \dot{e_x} \\ \dot{e_\lambda} \end{bmatrix} = A_{e,\sigma}(\rho)\begin{bmatrix} e_x \\ e_\lambda \end{bmatrix} - K_\sigma(.)(y-\hat{y}) + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w \quad (14)$$

Or equivalently:

$$\dot{e} = \begin{bmatrix} \dot{e_x} \\ \dot{e_\lambda} \end{bmatrix} = (A_{e,\sigma}(\rho) - K_\sigma(.)C_e)\begin{bmatrix} e_x \\ e_\lambda \end{bmatrix} + B_{1e}w \quad (15)$$

where $B_{1e} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$ and $K_\sigma(.)$ is the observer gain which has to be determined.
Problem definition: Let consider the switched LPV system (11). The system (13) is said to be an ESO for the system (11) if it satisfies the following conditions:

- when $w(t) \equiv 0$, the estimation errors (15) is asymptotically stable
- when $w(t) \neq 0$, the estimation error satisfies the following $\mathscr{L}_2$-induced gain performance criterion:

$$min \ \ \gamma \ \ s.t \sup_{w\neq 0, w\in\mathscr{L}_2} \frac{\| z \|_2}{\| w \|_2} < \gamma \quad (16)$$

where $\| \, . \, \|_2$ stands for $\mathscr{L}_2$ norm and z is given by:

$$\begin{cases} z = e & \text{if both state and fault estimation error are minimized} \\ z = [0 \ I_m]e & \text{if the fault estimation error only is to be minimized} \end{cases}$$

Therefore, the main problem now is to design the switched LPV observer (13). As mentioned previously, regarding to a switched observer, the main challenge is to ensure the stability requirement. To deal with, the main result presented in the the next section is a switched LPV observer with dwell-time. In order to ensure the stability of the switched observer, a piecewise linear in time Lyapunov function in quadratic form is used. The applied Lyapunov function is non-increasing at switching instants and is assigned seperately to each subsystem. During the dwell time, this function varies piecewise linearly in time and after the dwell time, it becomes time invariant. Such a Lyapunov function allows to derive the stability conditions for the switched LPV system. Moreover, the minimization of $\mathscr{L}_2$ induced gain is performed in order to minimize the effect of disturbance on the estimation error.

3 Preliminaries on the stability of switched LPV system

3.1 Recall for the LTI case

This section is devoted to recall some results on the stability analysis for the continuous time, switched system by using the multiple Lyapunov function. The interested readers can refer to several works of [13], [15], [9], [2].
Let us consider the following switched LTI system:

$$\dot{x}(t) = A_{\sigma(t)}x(t), \qquad x(0) = x_0, \quad (17)$$

where $\sigma(t)$ is the switching signal and $A_{\sigma(t)} \in \{A_1, ..., A_M\}$, $A_i \in \mathbb{R}^{n\times n}, i = 1...M$. Obviously, this model is discontinuous w.r.t $A_{\sigma(t)}$ since this matrix jumps instanteously from A_i to A_j for $i \neq j$.
It is shown in [9] that the stability of this switched LTI system is ensured under a minimum dwell time i.e, if, for some $T>0$, there exists a family of symmetric and positive Lyapunov matrices $\{P_1, ...P_M\}$, such that:

$$\begin{gathered} A_i'P_i + P_iA_i < 0, \quad \forall i = 1,...M \\ e^{A_i'T}P_je^{A_iT} - P_i < 0, \quad \forall i \neq j = 1,...M \end{gathered} \quad (18)$$

Then the system is globally asymptotically stable for a dwell time greater than or equal to T. However, since the condition (18) is non convex in A_i, it is not easy to generalize to a system with uncertainties or to an LPV system.
More recently, an alternative solution was proposed in [2] to deal with both nominal and uncertain systems using a piecewise linear in time Lyapunov quadratic function. This Lyapunov function is non-increasing at the switching instants

and is assigned to each subsystem. This method could be more conservative than the one presented in [9], however it provides an efficient way to deal with uncertainties and LPV systems. Inspired by this approach, one will use this kind of Lyapunov function in order to guarantee the stability of the switched LPV system in the next section.

3.2 Stability condition for switched LPV system

Now, the LPV case is taken into account with the following switched LPV system:

$$\dot{x}(t) = A_{\sigma(t)}(\rho)x(t), \qquad x(0) = x_0, \tag{19}$$

where $\sigma(t)$ is the switching signal, $\rho = [\rho_1\ \rho_2\ .\ .\ \rho_m]^T \in \Omega \subset \mathbb{R}^m$ is the scheduling parameter vector and $A_{\sigma(t)}(\rho) \in \{A_1(\rho), ..., A_M(\rho)\}$, $A_i \in \mathbb{R}^{n\times n}, i = 1...M$.

Let consider the a piecewise linear in time Lyapunov candidate function $V(x(t)) = x'(t)P_{\sigma(t)}x(t)$ where $P_{\sigma(t)}$ is given as:

$$P_{\sigma(t)}(t) = \begin{cases} P_{i,k} + (P_{i,k+1} - P_{i,k})\dfrac{t-\tau_{s,k}}{T/G} := \hat{P}_{i,k}, if\ t \in [\tau_{s,k},\ \tau_{s,k+1}) \\ P_{i,G} \qquad\qquad if\ t \in [\tau_{s,G},\ \tau_{s+1,0}) \\ P_{i_0,G} \qquad\qquad if\ t \in [0, \tau_1) \end{cases} \tag{20}$$

where $i = 1, ...M$ with M is number of subsystems, $i_0 = \sigma(0)$. $\tau_1, \tau_2, ...$ are the switching instants. T is the dwell time satisfying $\tau_{s+1} - \tau_s \geq T$, and $\tau_{s,k} = \tau_s + k(T/G)$ for $k = 0, ...G$, $\tau_s = \tau_{s,0}$, $\tau_{s,G} = \tau_s + T$). $P_{i,k}$ are symmetric matrices of compatible dimensions, where G is an integer that may be chosen a priori.

The switched LPV system (19) is asymptotically stable if all subsystems are stable and the Lyapunov function $V(x(t))$ is non-increasing at the switching instants, i.e:

$$A_i(\rho)'P_\sigma(t) + P_\sigma(t)A_i(\rho) + \dot{P}_\sigma(t) < 0, \forall i = 1, ...M \tag{21}$$

$$V(x(\tau_k)) \leq V(x(\tau_k^-)), \tau_k \text{ is the switching instant} \tag{22}$$

In order to transform these conditions into convex conditions, the following lemma is used:

Lemma 1 ([4]) *Assume that for some interval $t \in [t_0\ \ t_f]$, and $\delta = t_f - t_0$, there exist two symmetric positive matrices P_1, P_2 of appropriate dimensions that satisfy the following conditions:*

$$\frac{P_2 - P_1}{\delta} + P_1A + A'P_1 < 0, \quad \frac{P_2 - P_1}{\delta} + P_2A + A'P_2 < 0 \tag{23}$$

Then, for the system $\dot{x} = Ax$, the Lyapunov function $V(t) = x'(t)P(t)x(t)$, where $P(t) = P_1 + (P_2 - P_1)\dfrac{t-t_0}{\delta}$, is strictly decreasing over the interval $[t_0\ \ t_f]$.

Then, by using the formula of $P_\sigma(t)$ in (20) and applying the polytopic approach for the LPV system, the following theorem gives stability conditions for the switched LPV system (19):

Theorem 2 *Consider the switched LPV system (19), if there exists a collection of matrices $P_{i,k} > 0, k = 0, ...G$, $i = 1, ...M$, of appropriate dimensions and G is a prescribed integer, such that for all $i = 1, ..M$ and $j = 1, ...N$, ($N = 2^m$: number of the vertices of the polytope), the following LMIs hold:*

$$\frac{(P_{i,k+1} - P_{i,k})}{T/G} + A_i^{(j)'}P_{i,h} + P_{i,h}A_i^{(j)} < 0, \tag{24}$$

$$k = 0, ...G-1, h = k, k+1$$

$$A_i^{(j)'}P_{i,G} + P_{i,K}A_i^{(j)} < 0 \tag{25}$$

$$P_{i,G} - P_{l,0} \geq 0 \quad \forall l = 1, ..i-1, i+1, ...M. \tag{26}$$

then the switched LPV system (19) is asymptotically stable.

Proof: The switched LPV system (19) is asymptotically stable if:

$$\begin{cases} \dot{V}(x) = x(t)'(A_i(\rho)'P_\sigma(t) + P_\sigma(t)A_i(\rho) + \dot{P}_\sigma(t))x(t) < 0 \\ V(x(\tau_k)) \leq V(x(\tau_k^-)), \tau_k \text{ is the switching instant} \end{cases} \tag{27}$$

It is assumed that at switching instant τ_k, the system switches from $A_i(\rho)$ to $A_l(\rho)$, i.e we can write the Lyapunov function at instant τ_k as:

$$V(x(\tau_k^-)) = x(\tau_k)P_{i,G}x(\tau_k), \quad V(x(\tau_k)) = x(\tau_k)P_{l,0}x(\tau_k) \tag{28}$$

Then the non-increasing Lyapunov function condition holds if $P_{i,G} - P_{l,0} \geq 0$ which is actually (27).

Now, we apply Lemma 1 for each switching instant τ_s. Indeed, during the dwell time, considering the time interval $[\tau_{s,k}, \tau_{s,k+1})$, the Lyapunov matrix $P_\sigma(t)$ changes linearly from $P_{i,k}$ to $P_{i,k+1}$. Then from Lemma 1 with $\delta = \tau_{s,k+1} - \tau_{s,k} = T/G$ and the polytopic approach for LPV system, (27) is true if (24) holds.

After the dwell time and before the next switching instant, i.e $t \in [\tau_{s,G}, \tau_{s+1,0})$, the Lyapunov matrix $P_\sigma(t)$ becomes time invariant $P_{i,G}$, then (27) holds if $A_i(\rho)'P_{i,G} + P_{i,G}A_i(\rho) < 0$ that is equivalent to (25) thanks to polytopic approach for LPV system. □

Remark 1 *Such a choice of piecewise Lyapunov function as mentioned above gives an efficient way to deal with the stability analysis problem for the class of switched LPV system. Indeed, the Lyapunov matrix P depends on time but not on parameter $\rho(t)$, then the derivative of P depends on the varition of the piecewise Lyapunov function during the dwell time and not on $\dot{\rho}(t)$. An alternative solution is to use the Lifted Conditions with Lyapunov dependant parameter function to convexify the stability condition as in [6].*

Extension to the observation problem:

Consider the system (19) and $y = C_{\sigma(t)}(\rho)x \in \mathbb{R}^p$ is the output vector of the system. Then, the states of the system (19) are reconstructable if there exist a matrix $P_{\sigma(t)}(t) \in \mathbb{R}^{n\times n}$ of form of (20) and a function $K_{\sigma(t)}(t) \in \mathbb{R}^{n\times p}$, such that:

$$\begin{aligned} &P_\sigma(t)[A_\sigma(\rho) - K_\sigma(t)C_\sigma(\rho)] \\ &+[A_\sigma(\rho) - K_\sigma(t)C_\sigma(\rho)]'P_\sigma(t) + \dot{P}_\sigma(t) < 0 \end{aligned} \tag{29}$$

The following theorem allows to solve the problem in (29):

Theorem 3 *Consider the switched LPV system (19), if there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0, \ldots G, \quad i = 1, \ldots M$, of appropriate dimensions and G is a prescribed integer, such that for all $i = 1, ..M$ and $j = 1, \ldots N$, ($N = 2^m$: number of the vertices of the polytope), the following LMIs hold:*

$$\frac{(P_{i,k+1} - P_{i,k})}{T/G} + A^{(j)\prime}{}_{i}P_{i,h} - C_i^{\prime}Y_{i,h}^{\prime} + P_{i,h}A^{(j)}{}_{i} - Y_{i,h}C_i < 0, \quad k = 0, \ldots G-1, h = k, k+1 \tag{30}$$

$$A^{(j)\prime}{}_{i}P_{i,G} - C_i^{\prime}Y_{i,G}^{\prime} + P_{i,K}A^{(j)}{}_{i} - Y_{i,G}C_i < 0 \tag{31}$$

$$P_{i,K} - P_{l,0} \geq 0 \quad \forall l = 1, ..i-1, i+1, \ldots M \tag{32}$$

then the states of the system (19) are reconstructable. And $K_{\sigma(t)}(t)$ is given by:

$$K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1}Y_{\sigma(t)}(t) = \begin{cases} \hat{P}_{i,k}^{-1}\hat{Y}_{i,k}, \ if \quad t \in [\tau_{s,k}, \ \tau_{s,k+1}) \\ P_{i,G}^{-1}Y_{i,G}, \quad if \quad t \in [\tau_{s,G}, \ \tau_{s+1,0}) \\ P_{i_0,G}^{-1}Y_{i_0,G}, \ if \quad t \in [0, \tau_1) \end{cases} \tag{33}$$

where:

$$Y_{\sigma(t)}(t) = \begin{cases} Y_{i,k} + (Y_{i,k+1} - Y_{i,k})\dfrac{t-\tau_{s,k}}{T/G} := \hat{Y}_{i,k}, if \quad t \in [\tau_{s,k}, \tau_{s,k+1}) \\ Y_{i,G} \qquad if \ t \in [\tau_{s,G}, \tau_{s+1,0}) \\ Y_{i_0,G} \qquad if \ t \in [0, \tau_1) \end{cases} \tag{34}$$

.

Proof: The proof can be infered easily from the theorem 2.

4 Switched LPV observer under a dwell-time constraint

In this section, the design of the switched observer using the dwell time notion is presented.

Considering the switched system (11), as presented in section 2. The following switched LPV extended observer is proposed to estimate the system's state and the effectiveness coefficient vector λ :

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\lambda}} \end{bmatrix} = A_{e,\sigma}(\rho) \begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix} + K_{\sigma}(t)(y - \hat{y}) \tag{35}$$

$$\hat{y} = C_e \begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix}$$

and the estimation error $e(t)$ is calculated by:

$$\dot{e} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_\lambda \end{bmatrix} = (A_{e,\sigma}(\rho) - K_{\sigma}(t)C_e) \begin{bmatrix} e_x \\ e_\lambda \end{bmatrix} + B_{1e}w \tag{36}$$

where $B_{1e} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$ and $K_{\sigma}(t)$ is the observer gain which has to be determined.

Now, regarding to the **Problem definition** in section 2, and thank to the Bounded Real Lemma and the Polytopic approach, the following theorem allows to compute the observer gain.

Theorem 4 *Consider the switched system (11) and the switched extended observer (35). If there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0, \ldots G, \quad i = 1, \ldots M$, of appropriate dimensions and G is a prescribed integer, such that for all $i = 1, ..M$ and $j = 1, \ldots N$, ($N = 2^m$: number of the vertices of the polytope), the following LMIs hold:*

$$\begin{bmatrix} \dfrac{(P_{i,k+1} - P_{i,k})}{T/G} + He[P_{i,h}A^{(j)}{}_{e,\sigma} - Y_{i,h}C_e] & \star & \star \\ B_{1e}^{\prime}P_{i,h} & -\gamma^2 I & \star \\ I & 0 & -I \end{bmatrix} < 0 \tag{37}$$

for $k = 0, \ldots G-1$, $h = k, k+1$,

$$\begin{bmatrix} A^{(j)\prime}{}_{e,\sigma}P_{i,G} - C_e^{\prime}Y_{i,G}^{\prime} + P_{i,G}A^{(j)}{}_{e,\sigma} - Y_{i,G}C_e & \star & \star \\ B_{1e}^{\prime}P_{i,G} & -\gamma^2 I & \star \\ I & 0 & -I \end{bmatrix} < 0 \tag{38}$$

$$P_{i,G} - P_{l,0} \geq 0 \quad \forall l = 1, ..i-1, i+1, \ldots M. \tag{39}$$

then

$$K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1}Y_{\sigma(t)}(t) = \begin{cases} \hat{P}_{i,k}^{-1}\hat{Y}_{i,k}, \ if \quad t \in [\tau_{s,k}, \ \tau_{s,k+1}) \\ P_{i,G}^{-1}Y_{i,G}, \quad if \quad t \in [\tau_{s,G}, \ \tau_{s+1,0}) \\ P_{i_0,G}^{-1}Y_{i_0,G}, \ if \quad t \in [0, \tau_1) \end{cases} \tag{40}$$

is the gain of the extended observer (35) and the error estimation asymptotically converges to zero for a dwell time of T, where:

$$Y_{\sigma(t)}(t) = \begin{cases} Y_{i,k} + (Y_{i,k+1} - Y_{i,k})\dfrac{t-\tau_{s,k}}{T/G} := \hat{Y}_{i,k}, if \quad t \in [\tau_{s,k}, \tau_{s,k+1}) \\ Y_{i,G} \qquad if \ t \in [\tau_{s,G}, \tau_{s+1,0}) \\ Y_{i_0,G} \qquad if \ t \in [0, \tau_1) \end{cases} \tag{41}$$

Proof: Let $V(e(t)) = e^{\prime}(t)P_{\sigma}(t)e(t)$ be the Lyapunov candidate function for the estimation error system (36) where $P_{\sigma}(t)$ is a piecewise linear in time Lyapunov matrix defined as in (20). From the Bounded Real Lemma, the condition (16) is satisfied if the following condition holds:

$$\dot{V} + e^{\prime}e - \gamma^2 w^{\prime}w < 0 \tag{42}$$

i.e:

$$\begin{bmatrix} He[P_{\sigma}(t)(A_{e,\sigma}(\rho) - K_{\sigma}(t)C_e)] + \dot{P}_{\sigma}(t) & P_{\sigma}(t)B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{43}$$

From the formula of $P_{\sigma}(t)$ in (20), (43) is satisfied if

$$\begin{bmatrix} He[P_{i,h}A_{e,\sigma}(\rho)-Y_{i,h}C_e]+\dfrac{(P_{i,k+1}-P_{i,k})}{T/G} & \star & \star \\ B'_{1e}P_{i,h} & -\gamma^2 I & \star \\ I & 0 & -I \end{bmatrix} < 0 \tag{44}$$

holds for $h = k, k+1$, $i = 1,2...M$, $k = 0,...G-1$.
and

$$\begin{bmatrix} A_{e,\sigma}(\rho)'P_{i,G}-C'_eY'_{i,G}+P_{i,G}A_{e,\sigma}(\rho)-Y_{i,G}C_e & \star & \star \\ B'_{1e}P_{i,G} & -\gamma^2 I & \star \\ I & 0 & -I \end{bmatrix} < 0 \tag{45}$$

hold for $i = 1,2...M$.
The equation (44) guarantees that the Lyapunov function $V_\sigma(t)$ decreases and that (42) holds during the time intervals $t \in [\tau_{s,0}, \tau_{s,G})$. The LMIs (45) ensure that $V_\sigma(t)$ decreases and that (42) holds after the dwell time and before the next switching instant, i.e $t \in [\tau_{s,G}, \tau_{s+1,0})$.
From the definition of $P_\sigma(t)$, consider that at instant τ_k, that the system switches from the mode i to the mode l. To guarantee the Lyapunov function is non-increasing at the switching instants, we must ensure:

$$P_{i,G} - P_{l,0} \geq 0 \quad \forall l = 1,..i-1, i+1,...M. \tag{46}$$

Now, in order to resolve the LMIs in (44), (45), we apply the polytopic solution for the LPV system where the polytope is given by $\Omega_\rho = \begin{bmatrix}\underline{\rho} & \bar{\rho}\end{bmatrix}$ and we obtain the LMIs in (37), (38). ■

Extension with a decay rate on the convergence:
The next result extends the previous one, imposing a prefixed decay rate on the convergence of the estimation error.

Theorem 5 *Consider the switched system (11) and the switched observer (35). If there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0,...G,\ \ i = 1,...M$, of appropriate dimensions, G is a prescribed integer, and a positive scalar β such that for all $i = 1,..M$ and $j = 1,...N$, ($N = 2^m$: number of the vertices of the polytope), the following LMIs hold:*

$$\begin{bmatrix} \dfrac{(P_{i,k+1}-P_{i,k})}{T/G}+He[P_{i,h}A^{(j)}{}_{e,\sigma}-Y_{i,h}C_e]+2\beta P_{i,h} & \star & \star \\ B'_{1e}P_{i,h} & -\gamma^2 I & \star \\ I & 0 & -I \end{bmatrix} < 0 \tag{47}$$

for $k = 0,...G-1,\ h = k, k+1,$

$$\begin{bmatrix} A^{(j)\prime}{}_{e,\sigma}P_{i,G}-C'_eY'_{i,G}+P_{i,K}A^{(j)}{}_{e,\sigma}-Y_{i,G}C_e+2\beta P_{i,G} & \star & \star \\ B'_{1e}P_{i,G} & -\gamma^2 I & \star \\ I & 0 & -I \end{bmatrix} < 0 \tag{48}$$

$$P_{i,K} - P_{l,0} \geq 0 \quad \forall l = 1,..i-1, i+1,...M. \tag{49}$$

then $K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1}Y_{\sigma(t)}(t)$ is the gain of the extended observer (35) and the error estimation asymptotically converges to zero for a dwell time of T.
Proof: The proof is similar to the last cases and is omitted here for the simplification.
Proposition 1 *The design of the switched observer (35) can be performed by solving the following optimization problem:*

$$\begin{aligned} &\min_{P_{i,k},Y_{i,k}} \gamma^2 \\ &\text{subject to} \quad (47),(48),(49) \quad \text{and} \quad P_{i,k} > 0 \end{aligned} \tag{50}$$

By solving this optimization problem, one can derive $P_\sigma(t), Y_\sigma(t)$ and the observer gain is calculated by $K_\sigma(t) = P_\sigma(t)^{-1}Y_\sigma(t)$.

5 Numerical Exemples

In this section, two different numerical examples will be presented in order to illustrate the effectiveness of the proposed switched LPV observer. Firstly, the present approach is applied for a semi-active suspension system that is actually a single input-multiple output (SIMO) system where the actuator is the semi-active damper. Then, an example with a multiple inputs multiple outputs (MIMO) system is used to emphasize the interesting of the presented apporach.

5.1 Actuator fault estimation for semi-active suspension system

The simulation is performed in a small pilot SOBEN Car equipped with four electrorheological semi-active dampers (see Fig. 2) using the experimental data. Only the quarter car model is considered. In this model, the quarter vehicle body is represented by the sprung mass (m_s), the wheel and tire are represented by the unsprung mass (m_{us}). They are connected by a spring with the stiffness coefficient k_s and a semi-active damper. The tire is modeled by a spring with the constant stiffness coefficient k_t. As seen in the figure, z_s (respectively z_{us}) is the vertical displacement around the equilibrium point of m_s (respectively m_{us}) and z_r stands for the road profile.

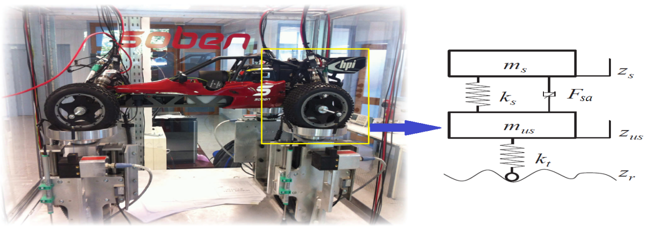

Fig. 2. Quarter-car vehicle model

The dynamical equations of such a quarter vehicle model are given by:

$$\begin{cases} m_s\ddot{z}_s &= -k_s(z_s - z_{us}) - F_{sa} \\ m_{us}\ddot{z}_{us} &= k_s(z_s - z_{us}) + F_{sa} - k_t(z_{us} - z_r) \end{cases} \tag{51}$$

where F_{sa} is the semi-active damper force.

The physical parameters characterize the quarter SOBEN car is given in the Table 1.

Table 1
Parameters of SOBEN car model

Parameters	$m_s[kg]$	$m_{us}[kg]$	$k_s[N/m]$	$k_t[N/m]$
Value	2.28	0.25	1800	12269

Let us consider now F_{sa} as the control input $u = F_{sa}$, and assume that a fault occured on the semi-active damper e.g an oil leakage which induces a lack of force modeled as:

$$\overline{F}_{sa} = \lambda F_{sa} = \lambda u \qquad (52)$$

where $\overline{F}_{sa}$ stands for the fault force expressed as a reduction of the nominal semi-active force and $\lambda \in [0\ 1]$ is the oil leakage degree, e.g. $\lambda = 0.8$ means that the damping force will be of 80% of the nominal damper force F_{sa} due to an efficiency loss of 20% .
Then the state space representation of the vertical dynamic using a quarter car model and taking into account a faulty semi-active damper, is given as follows:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 \lambda u \\ y &= Cx \end{aligned} \qquad (53)$$

where $x = (z_s,\ \dot{z}_s,\ z_{us},\ \dot{z}_s)^T \in R^4$ is the state vector, $w = z_r$ is the input disturbance, $u \in R$ is the control input, $y = [z_s - z_{us}, \dot{z}_s - \dot{z}_{us}, z_s] = [z_{def}, \dot{z}_{def}, z_s] \in R^3$ is the output vector and λ stands for actuator fault.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{m_s} & 0 & \frac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & 0 & -\frac{k_s+k_t}{m_{us}} & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_{us}} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{-1}{m_s} \\ 0 \\ \frac{1}{m_{us}} \end{bmatrix},$$

$$C = \begin{bmatrix} 1\ 0\ -1\ 0; 0\ 1\ 0\ -1; 1\ 0\ 0\ 0 \end{bmatrix}.$$

The semi-active damper force is assumed to be bounded by $-10N \leq F_{sa} \leq 10N$. Thus, the scheduling parameter $\rho(t) = |u(t)| = |F_{sa}|$ is considered to belong to $0.001 \leq \rho \leq 10$.
The quarter car model (53) is a SIMO system and can be rewritten in the form of the switched LPV system (10) where 2 subsystems $A_{e,1}(\rho), A_{e,2}(\rho)$ are given by:

$$A_{e,1}(\rho) = \begin{bmatrix} A & B_2\rho \\ 0_{1\times 4} & 0 \end{bmatrix}, A_{e,2}(\rho) = \begin{bmatrix} A & -B_2\rho \\ 0_{1\times 4} & 0 \end{bmatrix},$$

and $C_e = [C \quad 0_{3\times 1}]$
It can be easily obtained that the rank conditions in the Assumption A1 are satisfied:

$$rank\left(\begin{bmatrix} sI - A_{e,i}(\rho) \\ C_e \end{bmatrix}\right) = 5 \quad \text{for} \quad i = 1,2, \quad \text{all} \quad \rho \in \Omega = [0.001\ 10].$$

Then, the switched LPV observer is designed using a dwell time constraint as presented in the last section. With dwell time $T = 0.1s$, a prescribed integer $K = 1$, and a decay rate $\beta = 0.2$, by solving the optimization problem (50) with LMIs (47,48, 49) (where the decay rate is taken into account) for the 2 modes corresponding to $A_{e,1}(\rho), A_{e,2}(\rho)$ to obtain the matrices $P_{i,k}, Y_{i,k}, i = 1,2$ and $k = 0,1$. Then the gain of the switched observer is calculated by $K_\sigma(t) = P_\sigma(t)^{-1} Y_\sigma(t)$ where $P_\sigma(t), Y_\sigma(t)$ are given as in (20) and (41).
The road profile used in this test is a sinus profile as in Fig.3. The control input and the scheduling parameter are given in Fig.4.

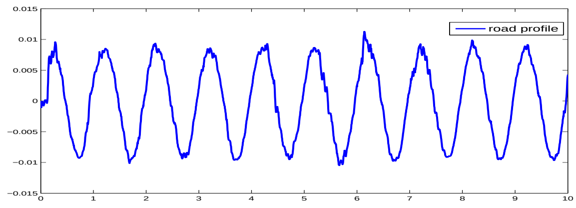

Fig. 3. Road profile

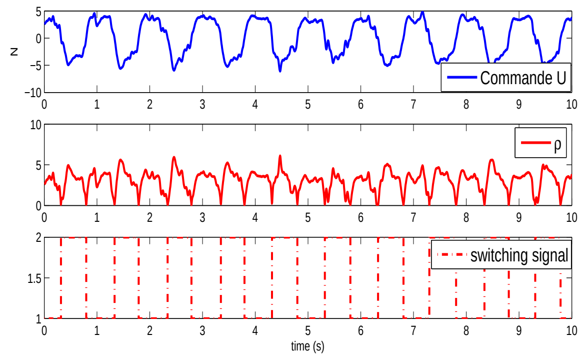

Fig. 4. The control input, varying parameter and switched signal

For the first scenario, it is assumed that an oil leakage occuring at t=5s causes a loss of 20% efficiency of damper force, the faulty damper remains 80% its healthy damper force (i.e $\lambda = 0.8$).

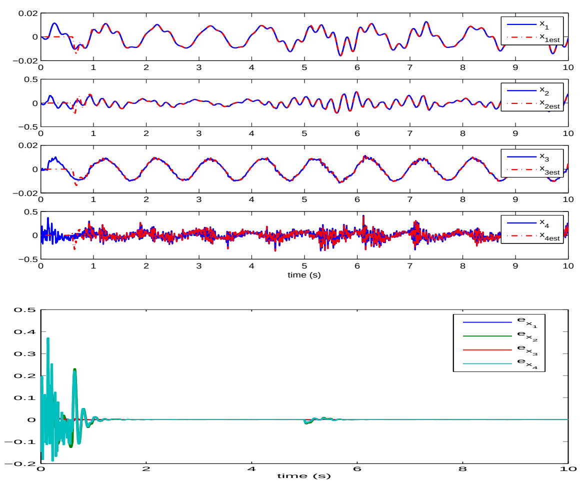

Fig. 5. State estimation (above) and error estimation

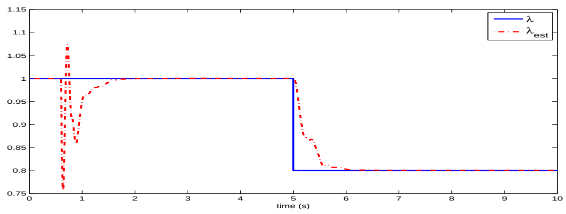

Fig. 6. A constant damper fault estimation

It is noted that the switched observer is activated from t=0.5s. Fig. 5 shows the state estimation and estimation errors. It can be seen that the state of the system is well estimated. Fig. 6 demonstrates that the switched LPV observer allows to estimate the effectiveness factor λ of the actuator.

Now, let us consider the second scenario where a gradual

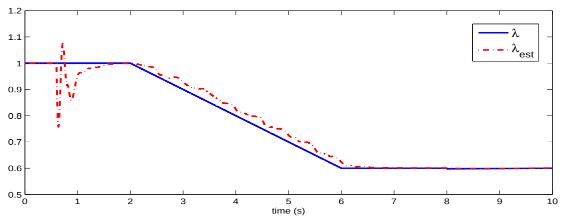

Fig. 7. A gradual damper fault scenario

fault is considered. The damper oil leaks slowly from t = 2-6s. The damper fault is well-estimated as plotted in Fig.7.

5.2 Actuator faults estimation for a MIMO system

Let us consider the following MIMO system:

$$\dot{x}(t) = Ax(t) + B_2\Lambda u(t) + B_1 w(t) \qquad (54)$$
$$y(t) = Cx(t)$$

$$A = \begin{bmatrix} -1.25 & 1 \\ 0.1 & -3 \end{bmatrix}, B_2 = \begin{bmatrix} 5 & 1 \\ 3 & 10 \end{bmatrix}, \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The system is subject to the control inputs $u_1(t) = 20\sin(4\pi t), u_2(t) = 30\sin(2\pi t)$ and the disturbance $w(t)$. Then, the scheduling parameters $\rho_1(t) = |u_1(t)|, \rho_2(t) = |u_2(t)|$ are assumed to be bounded by: $0.001 \leq \rho_1 \leq 20, 0.001 \leq \rho_2 \leq 30$. λ_1, λ_2 are the effectiveness factors of the 2 control inputs u_1, u_2.

We have 2 actuators, so the system (54) is rewritten as a switched system with 4 subsystems, i.e $A_{e,\sigma}(\rho)$ is switched between four modes $A_{e,1}(\rho), A_{e,2}(\rho)$, $A_{e,3}(\rho), A_{e,4}(\rho)$ according to the switching signal $\sigma(t)$ as follows:

$$\begin{cases} A_{e,\sigma}(\rho) = \begin{bmatrix} A & B_\sigma(\rho) \\ 0_{2\times 2} & 0_{2\times 2} \end{bmatrix}, \\ B_\sigma(\rho) \in B_2 \times \left\{ \begin{bmatrix} \rho_1 & \\ & \rho_2 \end{bmatrix}, \begin{bmatrix} -\rho_1 & \\ & \rho_2 \end{bmatrix}, \begin{bmatrix} \rho_1 & \\ & -\rho_2 \end{bmatrix}, \begin{bmatrix} -\rho_1 & \\ & -\rho_2 \end{bmatrix} \right\} \end{cases}$$

where ρ_1, ρ_2 belong to the polytope:

$\Omega_\rho = \left[\underline{\rho_1}\ \bar{\rho_1}\right] \vee \left[\underline{\rho_2}\ \bar{\rho_2}\right]$.

The Assumption A1 is also satisfied in this case. Then, in order to estimate the vector of effectiveness factors, the switched LPV observer is designed using the observer design procedure presented in Section 4.

Let us choose: $K = 1, T = 0.2, \beta = 1e-4$. By solving the optimization problem (50) with LMIs (47,48, 49) for the 4 modes corresponding to $A_{e,1}(\rho), A_{e,2}(\rho), A_{e,3}(\rho), A_{e,4}(\rho)$ to obtain the matrices $P_{i,k}, Y_{i,k}, i = 1,2,3,4$ and $k = 0,1$, then the gain of the switched observer is calculated by $K_\sigma(t) = P_\sigma(t)^{-1}Y_\sigma(t)$.

Fig. 8 shows disturbance $w(t)$, the control inputs u_1, u_2, the correspondant scheduling parmeters ρ_1, ρ_2 and the switching signal $\sigma(t)$.

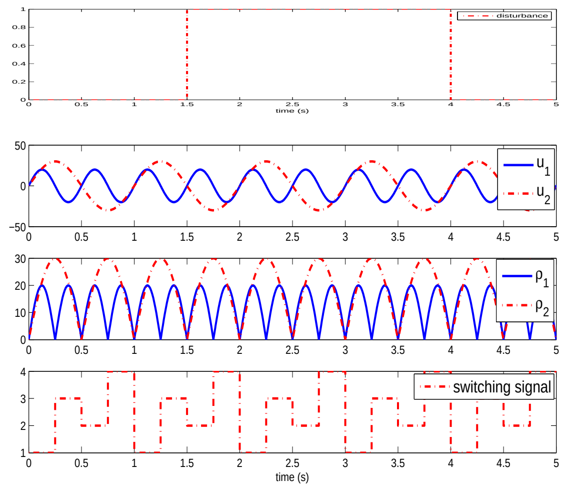

Fig. 8. Disturbance, control inputs, varying parameters, switching signal

Fig. 9 shows the estimation of the effectiveness factors λ_1, λ_2. Obviously, despite of the input disturbance $w(t)$, the switched observer allows to have a good estimation of the coefficients λ_1, λ_2.

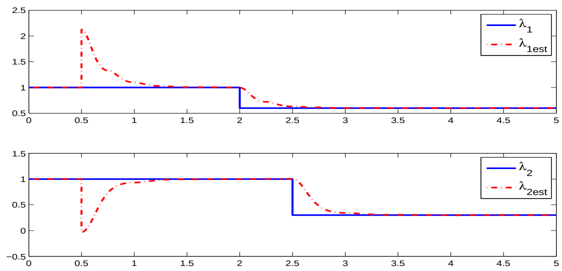

Fig. 9. The fault estimation result in a parital loss of effectiveness fault situation

6 Conclusion

In this paper, actuator fault estimation problem has been proposed within the LPV approach. The actuator faults are modeled in a multiplicative way by using the effectiveness factors ($\lambda_i \in [0 \quad 1]$). The fault estimation is based on a switched LPV observer. A non-increasing piece-wise linear time Lyapunov function is used to ensure the stability of the switched observer.The effectiveness of the proposed approach has been validated on a semi-active suspension system and an academic system. In the future work, some fault tolerant control strategies can be developped, based on the proposed fault diagnosis.

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